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No. 1811: February 20, 1918

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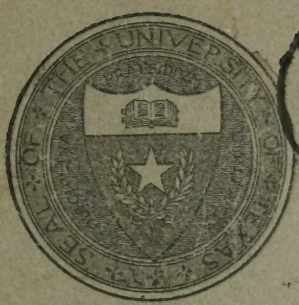
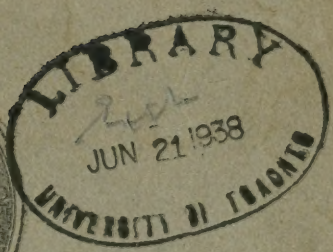
The Texas Mathematics Teachers' Bulletin

(Vol. 3, No. 2, February 20, 1918)





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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

University of Texas Bulletin

No. 1811: February 20, 1918

The Texas Mathematics Teachers' Bulletin

(Vol. 3, No. 2, February 20, 1918)

Edited by

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and

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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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MATHEMATICS TEACHERS' WEEK

In the summer of 1916, during the first four days of the University Summer School, there was held at the University of Texas a Mathematics Teachers' Conference. It was rather well advertised in this bulletin and in the literature sent out by the University concerning the session. About thirty teachers came for the conference. They were earnest and enthusiastic and there was a feeling that some good had been done. As the writer recalls the matter at this time, each teacher who attended undertook to come again the following year and bring with him two others.

Having this encouragement, a second conference for teachers of mathematics was planned for the same time in 1917. There had been some correspondence on the subject with teachers who had been at the first one, and it was the hope of those who had launched the movement that the next meeting would be both more numerously attended and more profitable to those attending than had been the first one. One of the hopes, at least, was not realized. The attendance instead of being larger than in 1916 was somewhat smaller; as to the comparative profitableness of the two occasions, the writer has no sure basis for a very positive opinion.

We find ourselves, therefore, in somewhat of a quandary as to whether to attempt a conference this summer. The days for the meeting would be June 12 to 15. Those of us who are at the University, especially those of us who started this movement, are loth to see the meetings discontinued, but have no mind to carry them on unless they can reach a reasonable number of teachers of mathematics in Texas Schools.

We have, accordingly, decided to put the matter up to the mathematics teachers themselves. If they want a conference and want it sufficiently numerously, and want it sufficiently earnestly, we will have one. Otherwise not. If the teachers of mathematics want a conference and will undertake to attend it we will prepare a program and undertake to give value received to those who come. We will assume the responsibility

for the character and worth-whileness of the conference, but you must undertake to come.

Now, gentle reader, the fate of Mathematics Teachers' Week is in your hands. You can perpetuate it or you can consign it to the limbo of things that have been. To those who favor its demise no instructions are necessary. "Silence is golden," also fatal. To those who wish to see mathematics week kept on the calendar and made a real force in the teaching of mathematics in Texas, we suggest the following line of conduct:

Write a letter to J. W. Calhoun, University of Texas, Austin, Texas, sometime before April 15, setting forth the following:

1. That you desire to see Mathematics Teachers' Week preserved, enlarged and improved.

2. That barring an act of Providence of considerable magnitude and seriousness, you will be on hand to attend and take part.

3. That you will speak of the conference to teachers of mathematics and try to induce them to attend.

Of these three things, by far the most important is number 2. You will not be counted as having voted "aye" unless you say you are coming. 'Faith without works is dead."

The result of this appeal and the fate of the conference will be announced in the Spring number of this Bulletin.

MATHEMATICS IN EVERYDAY LIFE

The mathematician does not attempt to justify mathematics, for himself, by its utilitarian value. Nor does a sculptor measure his art with that of a stone mason, nor a painter value his because, forsooth, pigments can be used to preserve weather boarding.

If the purpose of education is to enable men to make a living, utility should be the final test for the admission of any subject to a place in the curriculum; if, however, education is to fit men to live largely, utility is far from final. It is not my purpose now to review arguments in justification of the educators who for three thousand years have given mathematics a fundamental place in the course of study. The pith of these arguments, however, is spiritual rather than material, putting the enrichment of the mind above the feeding of the body.

Most of those who study mathematics, and most of us who teach it, are not in the strictest sense mathematicians, and we can afford to be rather patient with those who demand less exalted but more obvious justifications of mathematics. It is not unnatural that men and women should remark sometimes that they cannot see that their mathematics has ever done them any good. Perhaps it is the knowledge which we use upon occasion that most impresses us as knowledge; that used every day becomes commonplace and we forget that it is knowledge. We marvel at the skill of an acrobat who walks on his hands and forget the skill with which we have learned to walk on our feet. We admire the assurance of the pilot who steers us safe of hidden shoals, while we forget the confidence with which we thread the streets of our own city or the meandering roads and paths of our countryside.

Everyday mathematics so permeates the atmosphere of our every action that we forget that it is there, and we seem to do by instinct things that we learned to do at the outset with definite effort, such, for instance, as telling the time or making change.

The normal day in the normal life of the normal individual is full from its beginning to its end of mathematics at work.

The first thing which most of us do when we awake is to look at the clock. Fortunately, we do not have to recall each morning the evolution of systems of time and of time-pieces, nor do we stop to study the mathematical problems involved in the adjustment of springs and weights, the fitting of cog wheels, the effective shaping of escapements, the delicate compensation of pendulums and balance wheels.

But taking the clock at its word, reluctantly perhaps, we are up for a morning toilet. A ready faucet furnishes us water and the removal of a plug empties the basin. Back of this simple convenience is a whole system of watershed, intake, pumping station, water mains, sewage pipes, with numberless mathematical problems of natural drainage areas, dams, engines, size and strength of pipes, sewage flow and disposal. It takes little mathematics to open a faucet; it takes a great deal to make it worth while to do so.

The morning paper perhaps awaits us as we leave our rooms, and it is not at all necessary that we think each day of the problems of its make-up. But they involve lumbermen handling trees in the woods, rail and water transportation, pulp and paper mills, linotypes, and giant presses adjusted to the most delicate nicety. They involve, too, news services covering a world, with cables and telegraph and telephone, no one of which can be installed or maintained without the solution of a hundred mathematical questions.

Breakfast only increases our indebtedness, for its supplies may have come from the ends of the earth. And that means ships, with the mathematical problems of buoyancy, equilibrium, economy of carrying power; it means engines and power production and transmission, and propeller blades whose shape must be adapted to their purpose; it means guiding mechanism and a navigator's knowledge and skill; it means docks and derricks, built by plan and built with consideration of the strength of materials and the loads to be carried; it means charted seas and harbors, lighthouses and lenses; it means canals and locks; it means railroads, with the problems of cuts and fills and bridges, of curves and tangents and grades, of motive power and rolling stock, of schedules and rates; and it means fuel production, mine shafts, and galleries and tramways and hoists.

More yet, a good meal is a matter of food chemistry and of balanced diet, with its computations of proportions of proteins, fats, starch, sugar, and of energy values in calories.

Start now to your work, to school, or office, or store. You may be fortunate enough to own an automobile and to glide smoothly to work through the combined services of those who have solved the problems of oil production and refinement, of gas engines with their accurate adjustments, of rubber curing and tire making, of gearings and bearings. Or you may go by street car and trust yourself to the men who have solved the problems of electric motive power, whose mathematics is in units of ohm and watt and ampere and volt. Or, with better service to your health, you may walk, and enjoy the convenience of streets carefully graded, drained, paved.

Arrived at your post, if it is an indoor one, you have a new debt of gratitude to surveyor, architect, and sanitary engineer. If you date a letter, you probably refer to a calendar which it took hundreds of years to perfect and which astronomers devised though emperors and popes proclaimed. If you visit your bank you see everywhere the signs of precise reckoning, and if you buy or sell or trade you talk in terms of numbers. Even the clothes you wear have been patterned before they were cut.

But the question is inevitable whether these things are really everyday uses of mathematics. Of course they are. There is no waking or sleeping moment in which we are not served by utilities whose bases are essentially mathematical. Our contact with the minds which have provided these bases may be very remote or impersonal, but there must be such minds. Our cook may know nothing of the mathematics or chemistry of her kitchen, she may work solely by empirical rules and experience, but somewhere the problems have been worked out by something better than guess work. Our newsboy may be limited mathematically to penny calculations, but the intricate mechanical problems of the modern newspaper with adequate equipment have required skilled minds as well as skilled fingers. The chauffeur may run the car by faith, but engineers have toiled over blue prints and figures to make the car so nearly fool-proof. The motorman may operate his car wholly by rule but the problems of applied electricity require keen mathematical analysis.

The great General Electric Company with its thousands of employees pays a princely salary to Steinmetz, because it is Steinmetz with his mathematical genius who solves the problems that baffle the practical engineers.

When the American Bell Telephone Company sought two or three years ago to make it possible to talk across the continent, it put a small army of men to work. While physicists worked in their laboratories, mathematicians worked at their equations, and linemen and instrument makers took orders from both. Before the Atlantic cable there was Field, and behind the wireless operator at his key is the master mind of Marconi.

I have seen a clerk in a great store laboriously calculate the total of a short bill of goods, but in an inner room there was a man who could think in six figures and grasp the mathematics of the entire establishment.

The agent who goads you into sense enough to insure your life may know little about the policies he offers beyond what is tabulated for him in his little book—they all have one,—but sitting in an office at headquarters is an actuary who interprets mortality tables, estimates probabilities, computes premiums and risks, and sees to it that each new form of policy is founded on sound principles and that nothing short of pestilence can break his company.

Factories are manned by operatives skilled in a single step of a process which they have never known as a whole, but at a desk somewhere sits a man who is master of every detail, and who can talk in terms of cost sheets and time production, of transportation and markets, of profits and dividends.

The sailor on the sea may simply obey orders with no idea of route or destination, but we pin our faith and trust our lives to the man in the tiny cabin with his charts and compass and sextant and chronometers. And the metal worker who hammers rivets into the steel sides of the ship may be only an artisan, but the naval architect has figured out stresses and strains, displacement and tonnage, horse-power and speed before the keel is laid.

It must not be supposed that our indebtedness to mathematics is merely to simple processes of computation. Unfortunately that is all that mathematics means to many, while the fact is that some of the purely utilitarian uses of mathematics tax it

to the breaking point and call for the employment of some of its most recondite branches and the service of some of its most ingenious disciples.

All too sadly war has an everyday demand for mathematics now. The problems, for instance, of barrage fire are mathematical problems of shape and weight of projectiles, explosive charges, muzzle velocities, and trajectories. The aeroplane has been developed by a study of the behavior of surfaces subjected to motions in certain directions and at certain velocities with the resulting increases and decreases of air pressures. Safety in the air becomes possible only as solutions are found to problems of driving power, lifting power, and stability.

Of course I can hear the objection that many of the uses of mathematics here enumerated are in reality uses of physics and chemistry and engineering. But all of these sciences when they become exact become to that extent mathematical, and while a qualitative result may be suggestive, it is quantitative results that give to a science its practical application. All of the experimental sciences are striving to express themselves in mathematical language. Even biology, the science of the living organism, is seeking more and more a mathematical expression of its laws and is developing a new field of applied mathematics, biometry. And the social sciences, such as psychology, sociology, economics, education, not yet classed as exact sciences, approach such classification only to the extent to which they are able to express themselves in mathematical form.

Other uses of mathematics abound which are somewhat less obvious, but in the final analysis no less real. There are those to whom mathematics is the antithesis of art. But it is not difficult to show that art, in probably all its branches, pays conscious or unconscious tribute to mathematics. Not that art can be made a matter of equations or produced by a series of syllogisms. Certainly I should hate to think that the time would ever come when the genious of Beethoven could be reduced to formula or explained in terms of the shapes and dimensions of wood-wind and brass and stringed instruments.

It is nevertheless true that the basis of music is harmony and rythm and that both express themselves mathematically. A problem in harmony or counterpart is definitely a mathemat-

ical problem though it may be ever so much else besides. It is a very interesting observation that a musician who literally cringes before a problem whose elements are points and lines, or dollars and cents, tackles with eagerness a problem, no less mathematical, whose elements are tones and intervals. Be it remembered that mathematics is the method, the process, and not the elements themselves to which the method is applied. If it be argued that in music many results are possible, that there is no predetermined solution, I answer that equally so inventional geometry has no predetermined results, that its combinations are numberless. Old John Stuart Mill is said actually to have suffered because his computations told him that some day the possible musical combinations would be exhausted and creation become no longer possible—a rather needless ground for depression, for the sun will go cold before the combinations have all been found.

Painting and sculpture are modes of expression of the human spirit, but the painter must first of all be able to draw and the sculptor must first of all appreciate the form and proportions of the subject he is to portray. The ruler and calipers as well as the brush and chisel are tools of the studio.

Poetry, like music, is based upon sound values and rythm and in its mechanical terminology, which is inescapable, it even borrows words suggestive of its mathematical basis, such as length, meter, measure. Even such prose as lays claim to artistic quality has a sort of scansion of its own,—a mathematical form, if you will.

And this leads to the suggestion of a certain relationship between the study of mathematics and the use of language. The discussion as to the possibility of transferring discipline from one field into another is perhaps settled to the satisfaction of those only who consider it settled. For the rest of us it is still an open question. But whether the habit of careful thinking and accurate expression, which is inculcated by training in mathematics, whose language is perhaps the most coherent and compact ever invented, is or is not carried over into thought and expression in other subjects, it certainly must be true that the student who has once clearly perceived the possibility of rigorous thinking and unambiguous language must remain forever

conscious of their availability, though he may not use them. It would be unpardonably tedious did we put all our utterance into the precise language of demonstration, but we can never regret having learned that there is a distinction between assertion and proof.

Much of our knowledge and most of our actions are intuitive. That is what makes the beginnings of mathematics seem so useless to some, even as Euclid did to the immature Newton. We seem to be engaged in a much ado about nothing to prove what we already know. But in time comes the gleam of a new light that what we think we know is not always true, and that there are realms of knowledge in which we can test our intuitions to some extent at least. Mathematics does not make us wholly distrustful of our intuitions, but it does help to free us of unquestioning confidence in them, while it reassures us in the faith that there are processes of nature and of mind that can be depended upon, that there are some things of which we can be sure. It is only with the things of which we can be sure that mathematics, in the purest sense, deals at all.

Is the study of mathematics worth the while? To that there is only one possible answer. At every turn we are using it in one form or another, either consciously or unconsciously ourselves, or indirectly through the services of some one else. The world cannot do without it. Somebody must know it. The only question is as to who shall know it, to whom it shall be taught. And perhaps no one has had a full chance in life who has not had an opportunity to see whether or not he is one of those who can use it, for himself and for others. Doubtless, also, even those without any special aptitude should receive a minimum of training in mathematical method and should learn a minimum of mathematical fact. What that minimum should be it is outside the purpose of this paper to discuss. But we should be perhaps a little deaf to any claim that the acquiring of this minimum would prove irksome to some gentle soul. So is cutting teeth irksome, but we should not care to be a people of toothless gums. It is at least amusing that those who are seeking to belittle the place of mathematics in the secondary schools are trying to prove their case by figures. That the nature of the problem of mathematics in the curriculum is one

which does not admit of mathematical solution is of course evident. Massed statistics are imposing and suggestive, but no array of special instances ever suffices for proof of a general proposition.

The utilities of mathematics, far flung as they are, do not constitute its final claim upon the attention and interest of the human mind. It is truth for its own sake rather than because truth pays, which prompts the adventurous spirit in any domain. The realm of mathematics is in a sense an ideal world, and yet mathematics is no occult mystery; it is hidden from those who hide their eyes. If it seems to the uninitiated to be uncanny in its unerring accuracy, it is only because he does not realize that it knows its own limitations, that it is confident of its processes only and does not pretend to an accuracy in its results that is not present in its premises.

Because his realm is ideal, because the mathematician, when real conditions are unknown, sets up conditions of his own imagining, he becomes a creator and experiences something of the same intellectual and spiritual satisfaction which comes to artists and poets who create their own worlds. Mathematics is then its own reward.

HIGH SCHOOL ALGEBRA

"I never had any difficulty with algebra; it was always my easiest subject in high school."

How many teachers have heard this curious statement from pupils of low mathematical ability who could not understand plane geometry. These students know, and never knew, anything about their algebra except perhaps how to apply mechanically a few rules whose justification never occurred to them. These students have been drilled on a mysterious process of "transposing," "clearing of fractions," etc., and taught to find an *answer*. When asked the meaning of any of these steps or how they can verify their solution, they do not know what you are talking about. Not one in a hundred could tell you what is meant by a solution or root of an equation. When asked to solve the quadratic,

$$3\ (x-5)\ (x+4\tfrac{1}{2})=0$$

they will usually multiply it out, go through the process that has been taugh them, and may or may not arrive at the correct solutions. The difference between an identity and an equation has never been brought home to them, so that they cheerfully add fractions by first "clearing" them of fractions, especially if the fractions are literal. No wonder algebra never gave them any trouble, and they might also add that they never gave algebra any trouble.

If we look at the history of mathematical thought, the early and rapid development of geometry and the slow evolution of algebraic concepts; starting even in the case of the keen-witted Greeks, in the decadent age of Diophantus and only beginning to assume its modern aspect in the later middle ages, we can see why it is that algebra might be difficult for anyone—it is merely the history of human thought repeating itself. We can also see, perhaps, why when it is taught as a meaningless puzzle or game to be played according to certain rules, it might seem amusing and even interesting to sluggish minds that understood nothing of what they were doing and concluded that there was nothing to be understood.

From the educational point of view it is evident that this

method of teaching algebra is all wrong. It is worse than a waste of time to burden the memory of pupils with rules soon forgotten or changed into something "new and strange" (and wrong).

Of what possible use, either as a mental discipline or scientific tool could such training be? And yet the whole fabric of modern mathematics rests on algebra, and without a clear understanding of it no progress is possible, no unintelligent use of its rules is worth learning. Any one can learn how to throw an electric switch without the least knowledge of electricity, but mathematics must be understood to admit of useful applications. Mathematics as a tool is therefore a catch phrase capable of a very misleading interpretation.

To make algebra intelligible will mean that we must make it harder for those mentally too indolent to do more than memorize rules, but for those who must understand what they are about to be interested in it, it will prove far easier. The teacher's task will prove more exacting because it is more work to arouse intellectual exertion than to verify pure memoriter exercise.

Fortunately, it is possible to inject a large rational element into algebra by means of a few judicious omissions. If "clear of fraction," "transpose," and "cancel" are omitted from the bright vocabulary of youth and replaced by the processes of thought for which they should stand much will be gained. Every step should be made to depend on a well-understood axiom of quantity or definition or previous theorem. The rules for calculating with letters should be tested out on *numbers* and the habit of reckless induction should be controlled by careful experiment. The student who thinks that $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$, should be asked to try it out on numbers. The process of substituting numbers for letters should be abundant after the first few pages of the algebra are covered. Teach the student the reason for every step in solving a quadratic and make him see that when he is through with the process usually given, all he knows is that *if the quadratic has roots, we have gotten them.*

To transpose is to add the *same* number to both sides of an equation, *do not use the word transpose.* To clear of fractions is to multiply both sides of the equation by the common de-

nominator. To cancel is to realize that a number divided by itself gives one.

Many students do not fully understand the implications of the distributive and commutative laws, hence they *cancel* wrongly.

The rules $a(b+c+d) \equiv ab+ac+ad$,

$abcd \equiv adcb$, etc.

as well as $\frac{abcd}{e} \equiv \frac{a}{e} bcd$, etc.

Should be clearly brought out by the use of numbers or geometric diagrams.

Students who refuse to take things *on authority* have usually the best minds—they are not stubborn. The good teacher would do well to cultivate such intellectual recalcitrance and try to show them why they *must* accept as true his processes of calculation if they are reasoning beings.

THE LENGTH OF THE CIRCLE AND KINDRED TOPICS

In a long experience with teaching college freshmen it has never been the good fortune of the writer to find one who had the notion firmly fixed in his mind that length, area, and volume are numbers. When asked what is meant by the area of a rectangle they answer, "the amount of surface in it, the amount of space inside it, the room in it," or any one of half a dozen other vague or useless expressions. When asked what is meant by the length of a straight line segment they say: "How long it is." Asked to explain this answer they get back to the word length and are then just where they started.

It does not seem that either text or teacher places enough stress on the notion of a *unit of length* and its definition. The student should be made to understand clearly that the unit of length is arbitrary (so are nearly all other units), and that it belongs by definition to an arbitrary straight line segment. He should then have it clearly impressed on his mind that the length of any other straight line segment is the *number of pieces,* each congruent to the unit segment, into which the given segment can be cut. Length is in this sense a ratio, that is, a number. It is an abstract number (all numbers are really abstract). Length is given as 10 *miles,* 5 *feet,* 39 *kilometers,* etc., only to inform the reader or the hearer what unit was used. If we had only one unit of length no name would ever accompany a statement of the length of a segment. It would be 35, 63, 7½, etc., just as we say the age of a man is 21, 34, 47, 60, etc., without thinking it necessary to mention years.

If the student has his attention called specifically to this meaning of the word *length,* he will be able to see for himself that the expression "length of a circle" can have no meaning at all since the arc of a circle cannot be cut into straight line segments. He will accordingly see that either a circle cannot have length or that the definition of length must be extended so as to apply to the circular arc.

It is at this point that one impinges on the theory of limits, one of the bugbears of secondary mathematics. Now it is easy

to show that if a regular polygon (it is simpler, though not necessary, to use a regular polygon) is inscribed in a circle any regular polygon inscribed with twice as many sides will have a perimeter of greater length than the first, one with double the number of sides of the second a still larger perimeter, and so on. It is also easy to show that if a regular polygon is circumscribed about the circle, another circumscribed regular polgon with twice as many sides will have a perimeter of length less than the perimeter of the first circumscribed polygon. This process can also be continued. It is also easy to show that any circumscribed regular polygon has a perimeter of greater length than any inscribed regular polygon.

The length of the circle is now defined by making three assumptions.

1. The circle has length.

2. It is greater than the length of the perimeter of any inscribed regular polygon and less than the perimeter of any circumscribed regular polygon.

3. By taking the sides sufficiently numerous we can make the perimeters of an inscribed regular polygon and a circumscribed regular polygon, differ in length both from the circle and from each other by as small a number as we please.

Similar assumptions should now be made in regard to the area of a circle.

1. A circle has area.
2. The area of a circle is greater than the area of any regular polygon inscribed in it and less than the area of any regular polygon circumscribed about it.
3. By taking the sides of an inscribed regular polygon and a circumscribed regular polygon sufficiently numerous, we can make them differ in area from the circle and from each other by as little as we please.

This gives a meaning to the length and the area of the circle and does not mix the student much with the limit idea. It is now possible to give a simple proof of the theorem: "The area of a circle is equal to one-half the product of the lengths of the circumference and the radius."

Let P_c=perimeter of a circumscribed regular polygon.
P_i=perimeter of an inscribed regular polygon.
C=circumference of the circle.
R=radius of the circle.
R=apothegm of circumscribed polygon.
h =apothegm of inscribed polygon.
A_c=area of circumscribed regular polygon.
A_i=area of inscribed regular polygon.
A =area of circle.

Then $A_c=\frac{1}{2} P_c.R$
$A_i=\frac{1}{2} P_i.h$

$(P_c>C>P_i)$

And $\frac{1}{2} P_c.R>\frac{1}{2} C. R>\frac{1}{2} P_i.h$ (and $R>h$)

$(\frac{1}{2} P_c.R=A_c)$

Also $\frac{1}{2} P_c.R>A>\frac{1}{2} P_i.h.$ $(\frac{1}{2} P_i.h=A_i)$

$(A_c.>A>A_i)$

Now by taking the sides of the two polygons sufficiently numerous, $\frac{1}{2} P_c.R$ can be made to differ from $\frac{1}{2} P_i.h$ by as little as we please. But $\frac{1}{2}$ C.R and A will always be between them. Therefore, A and $\frac{1}{2}$ C.R canot be different, for if they were different they would differ by some fixed number, say K. If then we made $\frac{1}{2} P_c.R$ and $\frac{1}{2} P_i.h$ to differ by less than K then both A and $\frac{1}{2}$ C.R could not be between $\frac{1}{2} P_c.R$ and $\frac{1}{2} P_i.h$. But both A and $\frac{1}{2}$ C. R *always are between* $\frac{1}{2} P_c.R$ and $\frac{1}{2} P_i.h$. $\therefore A=\frac{1}{2}$ C.R q.e.d.

In solid geometry the problems of finding the lateral areas of cylinders, cones and frustums of cones as well as the volumes of those figures are usually attacked by means of inscribing and circumscribing prisms in the case of cylinders and inscribing or circumscribing pyramids and frustums of pyramids in the case of cones and their frustums. Having drawn the necessary figures and settled the necessary preliminary conditions an appeal is made to the theory of limits to deduce the necessary formulas.

But these can be deduced in the same simple way as the formula for the area of the circle, e. g., let us take the problem of finding the lateral area of a cylinder. (It might be mentioned that the treatment usually given as well as the present

treatment is logically correct only in case the curves used are circles, since in plane geometry the length of no curve except the circle is treated.)

Assumptions concerning the relations to each other of the prisms inscribed in and circumscribed about a cylinder as well as their relation to the lateral surface of the cylinder must be made analogous to the assumptions made on the length and area of a circle.

The following symbolism is convenient:

S_c=Lateral area of circumscribed prism.

S_i=Lateral area of inscribed area.

S =Lateral area of cylinder.

P_c=Perimeter of a right section of circumscribed prism.

P_i=Perimeter of a right section of inscribed prism.

P =Perimeter of a right section of cylinder.

R =apothegm of P_c

R =radius of P

h =apothegm of P_i

E =an edge of each prism and an element of cylinder.

Then $S_c = P_c.E$

$S_i = P_i.E$

But $P_c.E > P.E > P_i.E$ $\quad (P_c > P > P_i)$

$P_c.E > S > P.E.$ (By definition of lateral area of cylinder.)

But $P_c.E$ and $P_i.E$ can be made to differ by as little as we please.

$\therefore S = P.E$ q.e.d.

To find the volume of a cylinder we assume:

1. The cylinder has volume.

2. It is greater than the volume of any inscribed prism and less than the volume of any circumscribed prism.

3. By taking the lateral faces of each sufficiently numerous we can make a circumscribed prism and an inscribed prism differ in volume from each other and from the cylinder by as little as we please.

Let V_c=volume of a circumscribed prism.

V_i=volume of an inscribed prism.

V =volume of cylinder.

B_c=base of circumscribed prism.

B_i=base of inscribed prism.
B =base of cylinder.
H =attitude of each.
Then $V_c=B_c.H$
$V_i=B_i.H$
$B_cH>B.H>B_i.H$
and $B_cH.>V>B_i.H$
$\therefore$ V=B.H q.e.d.

INTRODUCTION TO STATISTICS—A NEW COURSE IN MATHEMATICS AT THE UNIVERSITY OF TEXAS

In view of the ever-widening applications of mathematics to human activity, it has been deemed wise to offer to freshmen an elementary course in the mathematics of statistics, for the spring term, *Mathematics 1 (h)*.

Those somewhat familiar with the literature of mathematical statistics know that a two-or-three-year course would not carry a student very far into theoretical statistics, even with calculus as a prerequisite. On the other hand, a mathematician familiar with the theory of probability and of statistics might have little skill in manipulating the actual data furnished by experience.

Courses in mathematical statistics cannot take the place of courses in economics, business, education, psychology, or biology, where recourse is had to the statistical method. But a student whose bent is primarily toward some social or biological science will be distinctly benefited by an intensive drill in the mathematical notions that underlie statistics, that powerful tool in modern research.

In a thirty-hour freshman course, no extensive field can be covered. The catalogue announcement is as follows: "*It treats of: averages—means, median, and mode—graphical representation, index numbers, frequency distributions, dispersion, and correlation. Supplemented by algebra: computation, approximation, limit of error, method of differences, interpolation, and probability.*"

But it is hoped that the course will not merely contribute toward a students' vocabulary of technical terms and his skill in mathematical manipulation, but will give him some *logical training.* Some fallacies that grow and stick in the human minds are hard to eradicate. A great many—not all—minds seem to distinguish between the two statements: "All horses are animals"; "all animals are horses." But we have only to increase slightly the complexity or abstractness of a statement like this to make a pair of converse statements indistinguishable to many. One of the chief by-products of a course in plane

geometry—with its loci problems and pairs of converse theorems—should be, as I view it, to enable a student to sense the logical fallacy of substituting a converse. Fallacies almost as pernicious are often involved in the deductions of immature or designing statisticians. For example, it is proved by an elaborate tabulation that ninety per cent of the coffee-drinkers of a certain large town are very poor, and the conclusion is drawn that coffee-drinking leads to poverty. It is overlooked that among those who do not drink coffee, one hundred per cent might be very poor, so that, so far as that town was concerned, the only possible rich men were coffee-drinkers.

In geometry, indeed in algebra, some importance attaches to a proportion, the equality of two ratios. *The abuse of statistics often lies in the presentation of a single ratio instead of an equality or inequality of two ratios;* and the inexperienced reader may be deeply impressed and completely fooled thereby.

It should be remembered that the subject of statistics is many-sided and that proficiency in mathematics, or even in logic, will not in itself make a statistician. Perhaps statisticians are born, not made. *But it seems reasonable to affirm that even an elementary drill in the rudiments of mathematical statistics will increase the efficiency of those who have some natural aptitude toward statistical research.*

The text-book that will be used this year is King's *Elements of Statistical Method*—a Macmillan book. The same publishers have recently published Secrist's *An Introduction to Statistical Methods*, a book with many interesting features. A more advanced text is that of G. Udny Yule, *Introduction to the Theory of Statistics*, Charles Griffin & Co., Limited, London. Mention should also be made of Bowley's *Elements of Statistics*, P. S. King & Son, London. A more theoretical text for advanced students is that of Arne Fisher, *The Mathematical Theory of Probabilities*, The Macmillan Co.

THE FACTORING OF POLYNOMIALS

The definition of factoring given in some textbooks on Algebra is so broad as to be absolutely meaningless. One of the well-known high-school texts gives the definition: "To factor an algebraic expression is to find two or more expressions which, when multiplied together, will produce the given expression." In this sense, every algebraic expression is a factor of every other algebraic expression, for we can write $P=Q(P/Q)$, no matter what P and Q stand for.

Usually the idea of factoring is limited to polynomials. To factor a polynomial means then to find two or more *polynomials* whose product is the given polynomial. For example, x^2+x has the factors x and $x+1$; x^2-y^2 has the factors $x+y$ and $x-y$. On the other hand, although

$$x^2+xy+y^2=(x+y+\sqrt{xy})(x+y-\sqrt{xy}),$$

we do not call $x+y+\sqrt{xy}$ and $x+y-\sqrt{xy}$ factors of x^2+xy+y^2, because they are not polynomials. A polynomial which can be factored in this sense into two polynomials, neither of which is a constant, is said to be *reducible;* otherwise it is *irreducible.*

The coefficients of a factor are not necessarily rational: thus x^2-2y^2 has the factors $x+\sqrt{2}y$ and $x-\sqrt{2}y$. Similarly x^2-a has the factors $x-\sqrt{a}$ and $x+\sqrt{a}$ if it be regarded as a polynomial in x alone, but is irreducible if we regard it as a polynomial in *two* variables x and a.

Frequently a narrower definition of reducibility is desirable. In dealing with real polynomials, for example, a polynomial such as x^2+1 may be naturally regarded as irreducible, since its factors $x+\sqrt{-1}$ and $x-\sqrt{-1}$ are complex. To distinguish this from the previous case, x^2+1 is said to be *irreducible in the domain of real numbers.* Similarly, if we are dealing with polynomials whose coefficients are rational, it may be convenient to regard x^2-2 as irreducible, since its factors involve the irrational number $\sqrt{2}$.

To take care of all such cases, we will introduce the concept

of a *domain of rationality,* which is very important in some branches of Algebra. A set of numbers is said to form a domain of rationality if the sum, the difference, the product, and the quotient (the divisor being different from zero) are also numbers of the set. A polynomial whose coefficients are in a given domain of rationality is said to be reducible in that domain if it can be expressed as the product of two other polynomials, neither of which is a constant, whose coefficients are numbers of the domain.

The simplest domain of rationality is that consisting of all the rational numbers; if a and b are two rational numbers, then obviously $a+b$, $a-b$, ab, and a/b (if $b\neq 0$) are also rational numbers. This is called the *natural domain,* and is denoted by $R(1)$, because all its numbers can be obtained from the single number 1 by series of rational operations: $1+1=2$, $1+2=3$, $1-1=0$, $0-1=-1$, $2\div 3=\frac{2}{3}$, etc. In the natural domain the polynomials x^2+x, x^2-y^2, $x^3-\frac{1}{8}$, etc., are reducible, while x^2-2y^2, x^2+1, etc., are irreducible.

Other simple domains can be obtained by taking a particular irrational number, as $\sqrt{2}$, and forming all possible numbers from it by series of rational operations. Thus from $\sqrt{2}$ we get $\sqrt{2}+\sqrt{2}=2\sqrt{2}$, $\sqrt{2}+2\sqrt{2}=3\sqrt{2}$, etc., $\sqrt{2}\div\sqrt{2}=1$, $1+1=2$, etc., whence we see that we can get any number of the form $a+b\sqrt{2}$, where a and b are rational. Conversely, any number of this domain (which we denote by $R(\sqrt{2})$) can be expressed in this form; for the result of a series of additions, subtractions, multiplications and divisions is a number of the form

$$\frac{p+q\sqrt{2}}{r+s\sqrt{2}}$$

where p, q, r, and s are rational, and if the denominator is rationalized by multiplying both numerator and denominator by $r-s\sqrt{2}$, this becomes

$$\frac{pr-2qs}{r^2-2s^2}+\frac{qr-ps}{r^2-2s^2}\sqrt{2},$$

which has the required form. It will be noted that $R(\sqrt{2})$ contains all the numbers of $R(1)$. This is true of every domain, for if a is any number of the domain, $a/a=1$ is also in the

domain, and hence all the numbers which can be obtained rationally from 1.

The polynomials

$$x^2-2y^2=(x+\sqrt{2}y)(x-\sqrt{2}y),$$

$$x^2+\sqrt{2}x-1=(x-\sqrt{2})(x+2\sqrt{2}),$$

$$x^2+(3-2\sqrt{2})x-1-\sqrt{2}=(x+1+\sqrt{2})(x+2-3\sqrt{2})$$

are reducible in $R(\sqrt{2})$, but not in $R(1)$. Similarly

$$x^2+2x-1=(x+1+\sqrt{5})(x+1-\sqrt{5})$$

is reducible in $R(\sqrt{5})$ but not in $R(\sqrt{2})$ or $R(1)$. The polynomial x^4+1 is irreducible in $R(1)$, but is reducible in $R(\sqrt{2})$, since

$$x^4+1=(x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1);$$

it is also reducible in $R(\sqrt{-2})$, since

$$x^4+1=(x^2+\sqrt{-2}x-1)(x^2-\sqrt{-2}x-1);$$

also in $R(i)$, where $i=\sqrt{-1}$, since

$$x^4+1=(x^2+i)(x^2-i).$$

The linear factors of x^4+1 cannot be obtained in any of these domains; to get these, we may use the domain $R(i,\sqrt{2})$, composed of all numbers which result from rational operations on the two numbers i and $\sqrt{2}$; in this we have

$$x^4+1=\left(x+\frac{1+i}{\sqrt{2}}\right)\left(x-\frac{i+1}{\sqrt{2}}\right)\left(x+\frac{i-1}{\sqrt{2}}\right)\left(x-\frac{i-1}{\sqrt{2}}\right)$$

More general domains of rationality can be constructed by taking any given set of irrational numbers a, b, c,....., and performing all possible rational operations on them; the resulting domain may be denoted by R(a, b, c,.....). Still more extensive is the domain composed of all real numbers. This cannot be constructed by rational operations on any finite number of quantities, but it is seen at once from the definition that the set of all real numbers is a domain of rationality. Finally, including among its numbers all the domains which we have named, is the domain of all complex numbers.

It is proved in the Theory of Equations that every polynomial in one variable can be expressed as a product of linear factors, i. e., that every polynomial in one variable is reducible in the domain of all complex numbers. This important theorem is not true for polynomials in two or more variables; thus

x^2+y^2+1, which is of the second degree, cannot be factored in any domain of rationality; this may be proved by writing

$$x^2+y^2+1=(ax+by+c)(lx+my+n)$$

and equating the coefficients on the two sides of the equation; the resulting equations in a, b, c, l, m, n are found to be inconsistent.

The problem of factoring a given polynomial is in general a difficult one; for if the polynomial contains more than one variable we cannot tell in advance whether there exist any factors, or what their degrees are if they do exist. Even if factors are known to exist, as is the case when the polynomial contains only one variable, by the theorem quoted above, it may be very difficult, or even impossible, to find them by purely algebraic methods. Hence only a few simple types of polynomials can be treated in an elementary course in Algebra.

GATESVILLE, TEXAS, Jan. 17, 1918.

Mr. J. W. Calhoun, Austin, Texas.

DEAR MR. CALHOUN: When I was at the State Teachers' Association, I attempted to give you my name to place upon the mailing list of *The Mathematics Teachers' Bulletin*, but you insisted that I write you a letter instead. Several numbers of the bulletin were sent to the school here last year and were handed to me. I filled out the slip as requested and mailed it to you, but have never received a number since doing so. I enjoyed some articles in those numbers very much, especially one upon the teaching of arithmetic. I am anxious to receive some more issues.

I have been doing some work in my beginner's class in geometry that has produced unusually good results. It is not new work, for I think you would class it as constructional geometry, and I and practically all other mathematics teachers have used that to some extent. However, I have been using it in a way that is a little different from any work that I have done heretofore or have ever seen done.

In a school the size of this, where the teaching force and equipment are limited, the pupil has little option as to the course of study that he takes. And you understand that the mathematics teacher is now under such fire from the other departments that he has to prove himself about the livest member of the faculty and then be styled useless and out-of-date. The criticism has become so common that the pupils have caught the fever, and about the first thing that a geometry teacher has to do is to explain to the pupils why he has the right to impose such socalled useless knowledge upon them. I have grown tired of explaining my position, and when about the third day of school one of the boys asked, "What is the use of studying geometry, anyway?" I turned the question back upon the class and asked that before the next recitation each one see if he could find some reason for studying geometry. Of course you understand that boys and girls of their age began to look about them for concrete illustrations of geometric figures. One pupil said that he saw geometry in the church window; another, that the school building was decorated with geometric constructions;

another, that geometry was used in constructing the floor out at the station, etc. One little girl said that her mother used geometry in piecing her quilts. Finally, one ventured to declare that he had found that geometry was the most artistic study in the curriculum. They became so enthusiastic over it that one pupil said, "If you will show me how to make a star, I think that I can draw something pretty also." I went to the board to show him and every pencil in the room "got busy." The next day at recess one of the girls said, "Miss Watkins, is there any way that I can draw a line so that it will be perpendicular at exactly the middle of another line?" I showed her how to draw the perpendicular bisector of a line in class the next day and again the pencils "got busy." In a few days a girl gave me a design for linoleum that she had drawn by means of geometric constructions. In a way it was poorly constructed, but I showed it to the class and praised it highly. As a result there came to me numerous requests to show them how to construct regular figures, etc., and since the pupils saw an immediate need for them they seized upon them at once. I now have a collection of designs, boxes, blotters, etc., decorated with circles, stars, polygons, and other geometric figures that is quite interesting to examine; and in addition, I have the best geometry class that I have ever taught. I have not required this outside work of them, have not assisted them by planning or suggesting designs, have only, at their request, made in class some of the fundamental geometric constructions, and yet every member of the class has made something for me. I find that they do the required work perhaps a little better than others have done it and in addition that this class which has studied geometry about four months, can now divide a line into extreme and main ration, construct a regular octagon, etc., more rapidly and more accurately than my other pupils could when they had completed geometry.

However, I do not lay claim to all the credit of having stimulated them to such unusual voluntary action, for there are some such excellent pupils in the class that they need little encouragement, and the fact that the brighter and more industrious

boy brings to class something that he has made, stimulates the slower one into action.

Some teachers who have seen the work have asked me for some samples to take back to their schools, and I thought that you might be interested in seeing some of it. Any way, I would like to know what you think of it. Perhaps it will not be anything new to you, for you may have seen the same kind and better work done in connection with other geometry classes. If you want to take the time and trouble to examine some specimen of it, I will send you some.

In conclusion, I must beg your pardon for writing you such a long letter, but you were responsible for it, for had you taken my name, as I requested, I would never have bothered you with all this. Sincerely,

MILDRED WATKINS.

VARIATION

In the last two numbers of the bulletin it has been shown that an elementary notion of the equation and the use of letters to represent numbers would help very much in the study of ratio and proportion. In the statement and solution of problems in those subjects no notion of the negative number is required and hence the work can be given easily to students below the high school grades. The use of letters for numbers in ratio and proportion leads easily to a notion of constants and variables and thus lays the foundation for the later use of functions and the graph. In arithmetic the early notion of the variable quantity is given under the subject of *Variation*. The notion of the variable is a growth, so to speak, in the mathematical development and cannot be given once for all in any one definition or illustration. What is given below in the way of definition is intended to be suggestive only. The skilful teacher of arithmetic with his class before him will have a better set of illustrations at hand.

CONSTANTS: A quantity that retains a fixed value throughout a discussion is called a *constant*. Thus a definite length or the numerical measure of that length is a constant. The quantities 5 acres, 10 bushels, 13 feet are constants. The numerical measures 5, 10, 13 of these quantities are therefore constants. Hence abstract numbers 3, 4, 8, etc., are constants.

VARIABLES: A quantity that is changing in value under certain conditions is called a *variable*. A flowing stream of water is continually changing the volume of water in the cistern into which it flows. The numerical measure of the volume is constantly changing and is also a variable. The distance through which a moving body travels and the numerical measure of the distance are variables.

The relation $y=2x$ shows that the number y is twice the number x. The number y also varies in such a manner that it is always twice x. The ratio

$$\frac{y}{x}=2$$

is another way of expressing the same relation.

The relation xy=2 shows that as x increases, y decreases. This can also be written.

$$y=\frac{2}{x}.$$

Direct Ratio: If *a* and *b* vary in value, but vary so that their ratio remains the same, we may write

$$\frac{a}{b}=r$$

where *r* is some constant number. In order that this ratio may be the same, any change in *a* must have the same rate of change as that of *b*. We express this fact by writing

$$\frac{ma}{mb}=r$$

where m is some number. When two numbers *a* and *b* are so related, *a* is said to vary *directly* as *b*.

Example: If c denotes the circumference of a circle and *d* the diameter, then

$$c=\pi d \text{ or } \frac{c}{d}=\pi \text{ where } \pi=3.1416$$

and, hence, c varies directly as d as the circle increases.

Example: When x varies directly as y we have

$$y=mx$$

where *m* is a constant. If now x=2½ when y=3½ we have

$$3\tfrac{1}{2}=2\tfrac{1}{2}m$$

$$m=\frac{7}{5}$$

and now the relation may be written

$$y=\frac{7}{5}x$$

A number of problems may be given here to illustrate direct ratio. Thus:

Example: When x varies directly as y find the relation between x and y

(1) If x=3 when y=4

(2) If x=5 when y=3, etc.

Example: The weight of a square cut from a sheet of iron of uniform thickness varies directly as the square of a side. When one side is 2 feet, the weight is 16 pounds. Find the weight when one side is 5 feet.

Inverse Ratios: When a and b are so related that

$$ab=r$$

where r is constant we may write

$$a=\frac{r}{b}$$

and hence have

$$ma=\frac{mr}{b}=\frac{r}{\frac{b}{m}}$$

which shows that when a is increased m times that b is decreased m times. Two quantities related in this way are said to vary inversely.

Example: When x and y vary inversely we have

$$y=\frac{r}{x}$$

where r is a constant. If now y=3 when x=5 we find

$$3=\frac{r}{5}$$

$$r=15$$

and hence the relations may be written

$$xy=15 \text{ or } y=\frac{15}{x}$$

From this relation we now find

when $x=1$, $y=15$,

when $x=2$, $y=\frac{15}{2}$,

when $x=3$, $y=5$, etc.

and in this way notice the variation in y is inverse to that of x. Practice of this kind will give the student in his early experience a good notion of the variable and at this point construction of simple graphs may be shown as a later development of variation.

Example: If a bicycle wheel 2 ft. 6 in. in diameter makes 256 revolutions in going a certain distance, how many revolutions will a wheel 4½ feet in diameter make in going the same distance?

A GLIMPSE AT EARLY COLONIAL ALGEBRA

(Reprinted from *School and Society*, Jan. 5, 1917.)

In many of our libraries in the east and in collections of Americana, it is not unusual to find notebooks of students of the colonial period which set forth in the careful script of the time the work accomplished in arithmetic under itinerant schoolmasters or in the academies of New England. To find a notebook in geometry is not so easy, for Euclid, although a recognized author, or rather a recognized subject, was taught only in the few colleges that had been founded here and there, and the number of notebooks, like the number of students, must have been relatively small. As to algebra, notebooks in this science are so rare that one seldom finds them even in large collections of colonial material, and hence it becomes a matter of interest to those concerned with the history of education and of early American mathematics when one is discovered.

It happens by a coincidence of circumstances that there now exists in the museum housed in the old jail in York village, Maine, a manuscript on algebra by Samuel Langdon, sometime president of Harvard College, written under the direction of Isaac Greenwood, for several years professor of mathematics and natural philosophy in the same institution. The manuscript was found some years ago in the old Sewell mansion in the village, known as Coventry Hall, one of the finest pieces of colonial architecture in this section of the country. The Langdons and Sewells had intermarried, and this manuscript had been handed down in the maternal line until it found place in the Sewell library. Such a rarity deserves more than passing mention, and educational interests make it proper that some record of its contents and of the circumstances of its preparation should be made.

It consists of seventy-five numbered pages, with two unnumbered pages forming a front cover and eighteen unnumbered pages at the end, the latter being blank except for a few unimportant notes on one leaf. The pages are 14.7 cm. by 18.7 cm., and the book consists of forty-eight leaves, three-fourths of one

leaf having been cut out by the author. On the front cover appears the inscription, "Samuel Langdon's Book. July 24, 1739," and on the reverse of this leaf are the words: "Algebra by Isaac Greenwood, M. A. Began July 25, 1739." A colophon reads: "Finished writing Algebra August 17, 1739. Algebrae Finis," showing that young Langdon was about three weeks in copying the notes of his work under Greenwood.

Before considering the manuscript itself, it is of historical interest to speak of the two men concerned in its production. Isaac Greenwood was born in Boston in 1702, was graduated at Harvard in 1721, and in 1727 was called by his alma mater to the newly created Hollis professorship of mathematics and natural philosophy, the second man in American to occupy such a chair. Two years later he published anonymously a work on arithmetic, the first to appear in the New World at the hand of a native American. He resigned his professorship July 13, 1738, a year before this manuscript was copied, having been "guilty of many acts of gross intemperance, to the dishonor of God and the great hurt and reproach of the society." Whether Langdon took the work before the summer of 1738, copying the notes in the summer of 1739, or took work with Greenwood after the latter's resignation of his chair, does not appear, and indeed is a matter of little importance. At any rate, Greenwood was only about thirty-six or thirty-seven years old when he aroused the interest of his promising pupil of fifteen to sixteen in the comparatively neglected science of algebra. He died in 1745, only forty-three years of age, a scholar as measured by his opportunities, an unfortunate victim of his appetites, but nevertheless a promoter of sound education.

Samuel Langdon was born at Boston in 1723. He received the degree of A. B. at Harvard in 1740, at the age of seventeen and a half years. Three years later he received the degree of A. M., in 1745 he was licensed to preach, and as a member of the colonial army he was present at the capture of Louisburg. In 1762 the University of Aberdeen conferred upon him the honorary degree of S. T. D., and in 1774 he became president of Harvard. The academic crown was far from being an easy one in his day, however, at least for a fighting patriot of the

colonies, and in 1780 his resignation was forced by the wealthy loyalists upon whom the college was compelled to depend for a certain amount of its support. He died in 1797 at the age of seventy-four years.

We see, therefore, that the manuscript now preserved at York village was written at the close of Langdon's junior year, but that it is a record of work taken under Greenwood probably a year or two earlier. At any rate it represents the algebra taught at Harvard about 1737-38, to a boy of fifteen or sixteen, by one of the first men to hold a special chair of mathematics and natural philosophy in an American college.

The opening paragraph shows the influence of John Wallis, at least indirectly:

"This science is called algebra from two or three works in ye Arabian language, which may be interpreted either ye Art of Restitution and Comparison, or ye Art of Resolution and Equation. It is also known by various other names. The first yt wrote on this subject in Europe, termed it ye Rule of Restitution and Opposition. Since, it has been called by some, ye Analytick Art; by others Specious Arithmetick, *Regula rei et Census*, ye great art, modern geometry, universal mathematicks, etc."

The topics treated, expressed in the words and form of the manuscript, are as follows:

Notation, Algebraical Characters, Addition of Integers, Substraction, Multiplication of Algebraick Integers, Division, Algebraick Fractions, Addition and Substraction of Fractions, Multiplication of Fractions, Division of Fractions, Involution of whole Quantities, Fractional Involution, Evolution of whole Quantities, Fractional Evolution, Binomial Quantities, Involution, Involution of Binomial Fractional Quantities, Multinomial Quantities, Fractional Compound Quantities, Evolution of Multinomial Quantities, Surd Quantities, Notation, Reduction of Surds, Multiplication of Surds, Division of Surds, Addition and Substraction of Surds, Compound Surds, Equations, Reduction of Equations, Reduction by Addition, Reduction by Substraction, Reduction by Multiplication, Reduction by Division, Reduction by Involution, Reduction by Evolution, Reduction by

Analogies to Equations, and e Contra, the Method of resolving Algebraical Questions, General Rules concerning ye Reduction of Equations, Simple Equations, Mr. Oughtred's method of solving adfected Quadratics, The Solution of adfected Equations by taking away the second term, the Solution of Adfected Quadratic Equations, by ye Method of compleating ye square, Questions producing adfected Quadratick Equations, The Resolution of Cubick Equations, Cubick Equations by Substitution, Cubick Equations by Tryals & depression, The Solution of irregular Cubicks, The method of converging Series and Approximation. I of Simple Roots, II Adfected Equations, Mr. Ralphson's Theorems, Dr. Halley's method for solving equations of all sorts, Concerning ye method of solving Geometrical Questions Algebraically.

There are various interesting passages in the manuscript, a few of which are as follows:

The letters used by Algebraists are generally small Letters, because ye Capital are generally imployed in Geometrical figures. (Page 2.)

And such quantities as are negative are markt with ye Sign (—): thus $x—u$, & $u—x$ express an Affirmative quantity, as an Estate which may be noted by x, lessened by another quantity u, which denote ye debt thereof, being a negative quantity. (Page 3.)

Mr. Oughtred's method of solving adfected Quadratics . . . Rule Multiply ye absolute by 4, and add thereto ye square of ye Differential quantity: & ye Square Root of this.—ye Differential Quantity, being divided by 2 is ye quantity sought. (Page 43.)

Among the symbols, the bar is the only sign of aggregation used: ÷—÷ is used for continued proportion;] and [are used for greater than and less than, as in the seventeenth century works; $a \overset{\smile}{—} b$ is used for the difference between a and b, as is still occasionally done in elementary books; and although the second power of a is indicated by a^2, the more common form is aa, and on occasion there is found $aaaaa$ for a^5, and similarly for other letters. The symbols are mentioned as "first introduced by Vieta," which of course was far from being the case.

[In finding powers of a binomial] if ye numeral figures or Coefficients could be found, ye whole might be performed without multiplication; & this is done by ye following Rule, given by Sr. Isaac Newton. See p. 139. To find ye Co-efficients in Binomial Quantities.—Rule. Multiply ye Co-efficients into ye Index, & divide ye product by ye number of Terms, counting from ye left hand, & ye Quotient will be ye Co-efficient, or numerical figure of ye next successive Quantity. (Page 18.)

Irrational Quantities are noted thus $\surd$: 2, which is 2 with ye Sign of irrationality before it. (Page 22.)

When at any time there is ye Sign of Irrationality prefixt to mixt numbers, with ye Line of Separation over them as $\sqrt[3]{:7+\sqrt{:2}}$, it is called an universal Root. (Page 22.)

At the end of the manuscript, on the inside of the last cover leaf, the ratio of the circumference of a circle to the diameter is given correctly to thirty-two decimal places, but the symbol π is not used.

The existence of such a manuscript suggests the possibility of finding similar material relating to algebra, geometry, natural philosophy, conics, and even the calculus. It might well repay a student for his efforts in searching the small as well as the large museums, libraries, and collections of Americana with this simple purpose in view.

The little work has also a further suggestion to us in the disturbed condition of our present education, namely, that this boy of fifteen, a high-school youth as he would be today, was not bothering with hundreds of examples in factoring and in complicated fractions, but was giving his energies to the study of those cubic and numerical higher equations needed in his work in what was known as natural philosophy. Might not a certain part of the objection to our algebra of the present be removed if we gave less attention to various topics of a purely mechanical nature and more to the real applications of the science?

David Eugene Smith,
Teachers College, Columbia University.

WHAT GREAT MEN SAY ABOUT MATHEMATICS

The history of mathematics is important also as a valuable contribution to the history of civilization. Human progress is closely identified with scientific thought. Mathematical and physical researches are a reliable record of intellectual progress.

CAJORI, F.

Every mathematical book that is worth reading must be read "backwards and forwards," if I may use the expression. I would modify Lagrange's advice a little and say, "Go on, but often return to strengthen your faith." When you come on a hard or dreary passage, pass it over; and come back to it after you have seen its importance or found the need of it further on.

CHRSTAL, GEORGE.

A *general course* in mathematics should be required of all officers for its practical value, but no less for its educational value in training the mind to logical forms of thought, in developing the sense of absolute truthfulness, together with a confidence in the accomplishment of definite results by definite means.

ECHOLS, C. P.

Mathematics is the study which forms the foundation of the course (West Point Military Academy). This is necessary, both to impart to the mind that combined strength and versatility, the peculiar vigor and rapidity of comparison necessary for military action, and to pave the way for progress in the higher military sciences.

CONGRESSIONAL COMMITTEE ON MILITARY AFFAIRS.

THE STRAIGHT EDGE

If there is a weak-kneed brother among us who thinks maybe after all mathematics is "no count" let him read the second article in this Bulletin and put in the next week being ashamed of himself.

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Mathematics Teachers' Week! Thumbs up or down?

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Some of you people who grieve over the fact that children take no interest in mathematics, read the letter on page 29 and then go to your study and take stock.

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Mathematics Teachers' Week! Those in favor say "aye," also "me," likewise "coming."

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If you regard the theory of limits as "the limit" read the article on page 18 and see if you get any help.

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If anything in this Bulletin seems to you to be either good or bad enough to talk about tell us what you got or failed to get from looking at it.

YOU KNOW

The army is taking a large number of men from high school positions. These places must be filled chiefly by women. They will be filled by the best trained women.

Some women who see this will see the point and proceed to prepare to answer the call when it comes. Preparedness is as essential to an individual as to a nation.

The University of Texas Summer School will run a full quarter this year, June 12 to August 31. It will be divided into two complete terms of equal length. Regular university courses will be given. The University has more calls for prepared teachers every year than it can fill. Good teachers who want to be better teachers are especially urged to attend the summer session. Only one matriculation fee of five dollars whether you attend one or both terms.

University Summer Normal during first term, June 12 to July 25.

For information write

E. J. Mathews, Registrar,
Austin, Texas.

A STORY WITH A MORAL

In June, 1905, John Doe and Richard Roe graduated from the same high school. Each got a first-grade certificate and began to teach in the rural schools of the State in the fall of 1905.

John Doe's Career

1. Taught two years at fifty dollars a month.
2. Attended summer normals and got a permanent certificate.
3. Taught five years as principal of a village school at from sixty to ninety dollars a month.
4. Taught five years as superintendent of village schools, at an average salary of one thousand dollars a year.
5. Is now thirty years old and finds competition with college-trained teachers making it increasingly hard for him to secure advancement of any sort.

Richard Roe's Career

1. Taught two years at fifty dollars a month.
2. Attended University of Texas Summer School 1906-10.
3. Taught during this time at from seventy-five to one hundred dollars a month.
4. Spent 1910-12 at the University of Texas.
5. Finished his degree work by summers and correspondence, 1912-15, while superintendent at fifteen hundred dollars a year.
6. Took his B. A. in 1915.
7. Superintendent, 1915-16, at two thousand dollars a year; 1916-17, at twenty-four hundred; elected for term of 1918 at twenty-five hundred dollars.
8. Finds calls coming to him and positions opening without any solicitation.

Which of these two is more useful to the community in which he lives? Which career will you choose?

The University of Texas summer session runs this year June 12 to August 31. Two complete terms of equal length. Regular University work. One registration fee of five dollars entitles you to attend one or both terms.

Summer Normal during the first term.

For information address

E. J. MATHEWS, REGISTRAR,
AUSTIN, TEXAS.